Dr. B. B. HEGDE FIRST GRADE COLLEGE , KUNDUPURA.

MATHEMATICS ASSIGNMENT

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III BSc

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Linear Transformations

Proposition:- Let V be an n dimension vector space over F. Then V is isomorphic to the space $F^{(n)}$

Proof:- Let $V' = F^{(n)}$ and

Let $(e_1, e_2, \dots, e_n)$ be a basis of V over F then every $v \in V$ can be uniquely expressed as

$$v = a_1 e_1 + a_2 e_2 + \dots + a_n e_n \quad , a_i \in F$$

Define a mapping $T: V \to V'$ by $T(v) = (a_1, a_2, \dots, a_n)$

Let $W = b_1 e_1 + b_2 e_2 + \dots + b_n e_n$ then

$T(W) = (b_1, b_2, \dots, b_n)$

Consider, $T(v+W) = (a_1+b_1, a_2+b_2, \dots, a_n+b_n)$

$= (a_1, a_2, \dots, a_n) + (b_1, b_2, \dots, b_n)$

$T(v+W) = T(v) + T(W)$

and $T(av) = (aa_1, aa_2, \dots, aa_n)$

$= a(a_1, a_2, \dots, a_n)$

$= aT(v)$

$\therefore$ clearly T is a L.T

Also T is 1-1 because the expression $v = \sum_{i=1}^{n} a_i e_i$ is unique

Now for any $(a_1, a_2, \dots, a_n) \in F^{(n)}$ if we define

$$v = \sum_{i=1}^{n} a_i e_i \quad \text{then} \quad T(v) = (a_1, a_2, \dots, a_n)$$

Thus T is onto

$\therefore$ T is an isomorphism of V onto V'

$\therefore$ V is isomorphic to the space of $F^{(n)}$

Corollary:- T is an isomorphism if $m(T)$ is non singular

Proof:- If T is an isomorphism then T is non singular.
Then there exists some $T' \in L(V, V')$ such that $TT' = T'T = I$

$$m(TT') = m(T'T) = m(I) \quad \text{(by known property)}$$

$$m(T') \cdot m(T) = m(T) \cdot m(T') = I_n$$

$\Rightarrow$ $m(T)$ is non singular and its inverse is $m(T')$

Conversly

Let $m(T)$ be non singular then there exists an $n \times n$ matrix A such that

$$m(T) \cdot A = m(T) = I_n$$

If T_A is the Linear Transformation associated to A then
$m(T_A) = A$ so that

$$m(TT_A) = m(T_A) \cdot m(T) = A \cdot m(T) = I_n$$

i.e $\quad m(TT_A) = m(I)$

$\Rightarrow \quad TT_A = I \qquad [\because T \to m(T) \text{ is } 1\text{-}1]$

Similarly $\quad m(T_A T) = m(I)$

$\Rightarrow \quad T_A T = I$

Hence T is non singular which means T is an isomorphism.

1^{st} Isomorphism Theorem:-

Let $T : V \to V'$ be a Linear Transformation of V onto V' and let $W = \ker T$ then V/W is isomorphic to V' $[V/W \cong V']$

Proof:- Consider the mapping $\bar{T} : V/W \to V'$ defined by

$$\bar{T}(v+W) = T(v)$$

$\bar{T}$ is well defined because if $v_1 + W = v_2 + W$ then

$$v_1 - v_2 \in W = \ker T$$

ie $$T(v_1 - v_2) = 0$$

$$T(v_1) - T(v_2) = 0 \quad [\because T \text{ is L.T}]$$

Also $\bar{T}$ is a L.T because let $\bar{v_1} = v_1 + W$

then $\bar{T}(v_1 + W) = \bar{T}(\bar{v_1}) = T(v_1)$

$\bar{v_2} = v_2 + W$ then $\bar{T}(v_2 + W) = \bar{T}(\bar{v_2}) = T(v_2)$

Consider, $a\bar{v_1} + b\bar{v_2} = a(v_1 + W) + b(v_2 + W)$

$$= (av_1 + bv_2) + W$$
$$= \overline{av_1 + bv_2}$$

$$\therefore \bar{T}(a\bar{v_1} + b\bar{v_2}) = \bar{T}(\overline{av_1 + bv_2})$$

$$= T(av_1 + bv_2) \quad [\because \bar{T}(\bar{v_1}) = T(v_1)]$$

$$= aT(v_1) + bT(v_2) \quad [\because T \text{ is L.T}]$$

$$\bar{T}(a\bar{v_1} + b\bar{v_2}) = a\bar{T}(\bar{v_1}) + b\bar{T}(\bar{v_2}) \quad [\because \bar{T}(\bar{v_1}) = T(v_1) \text{ \& } \bar{T}(\bar{v_2}) = T(v_2)]$$

$\therefore \bar{T}$ is a L.T

$\bar{T}$ is 1-1 because $\bar{T}(\bar{v_1}) = 0$ then $T(v_1) = 0$

$\therefore v_1 \in W = \ker T$

$\Rightarrow v_1 \in W$ iff $\bar{v_1} = 0$

Clearly $\bar{T}$ is onto, since T is onto

hence $\bar{T}$ is an isomorphism

$\therefore V/W \cong V'$

Proposition:- Let V be a Vector Space over F and W be a subspace of V. Let $v_1, v_2, \dots, v_n$ be a basis of V such that $v_1, v_2, \dots, v_m$ (where $m \leq n$) is a basis of W then $\bar{v}_{m+1}, \bar{v}_{m+2}, \dots, \bar{v}_n$ is a basis of V/W over F.

Proof:- Let $\bar{v} = v + W \in V/W$ be an element of V/W for any $v \in V$, $v = \sum_{i=1}^{n} a_i v_i$

then $v + W = (a_1 v_1 + a_2 v_2 + \dots + a_n v_n) + W$

$= a_1(v_1 + W) + a_2(v_2 + W) + \dots + a_m(v_m + W) + a_{m+1}(v_{m+1} + W) + \dots + a_n(v_n + W)$

$= a_1 \bar{v}_1 + a_2 \bar{v}_2 + \dots + a_m \bar{v}_m + a_{m+1} \bar{v}_{m+1} + \dots + a_n \bar{v}_n$

$v + W = a_{m+1} \bar{v}_{m+1} + \dots + a_n \bar{v}_n$ $\quad$ ($\because v_1, v_2, \dots, v_m$ is a basis of W)

$\Rightarrow v_1, v_2, \dots, v_m \in W$

iff $\bar{v}_1 = \bar{v}_2 = \dots \bar{v}_m = \bar{0}$]

$\therefore \{\bar{v}_{m+1}, \bar{v}_{m+2}, \dots, \bar{v}_n\}$ generate V/W

To prove that $\bar{v}_{m+1}, \bar{v}_{m+2}, \dots, \bar{v}_n$ are L.I

consider $b_{m+1} \bar{v}_{m+1} + b_{m+2} \bar{v}_{m+2} + \dots + b_n \bar{v}_n = \bar{0}$

$b_{m+1}(v_{m+1} + W) + b_{m+2}(v_{m+2} + W) + \dots + b_n(v_n + W) = \bar{0}$

$(b_{m+1} v_{m+1} + \dots + b_n v_n) + W = \bar{0}$

$\overline{b_{m+1} v_{m+1} + \dots + b_n v_n} = \bar{0}$

iff $b_{m+1} v_{m+1} + \dots + b_n v_n \in W$ $\quad$ [since $v \in W$ iff $\bar{v} = \bar{0}$]

i.e $\sum_{i=m+1}^{n} b_i v_i \in W$

Since $v_1, v_2, \dots, v_m$ is a basis of W

$$\therefore \sum_{i=1}^{m} c_i v_i = \sum_{i=m+1}^{n} b_i v_i$$

$$\sum_{i=1}^{m} c_i v_i - \sum_{i=m+1}^{n} b_i v_i = 0$$

$$c_1 v_1 + \dots + c_m v_m - b_{m+1} v_{m+1} - \dots - b_n v_n = 0$$

$$c_1 v_1 + \dots + c_m v_m + (-b_{m+1}) v_{m+1} + \dots + (-b_n) v_n = 0$$

But $v_1, v_2, \dots, v_n$ are the basis of V

$\therefore$ $v_1, v_2, \dots, v_n$ are linearly independent.

$$\therefore c_1, c_2, \dots, c_m = -b_{m+1} = \dots = -b_n = 0$$

$$\Rightarrow b_{m+1} = b_{m+2} = \dots = b_n = 0$$

$\therefore$ $\overline{v_{m+1}}, \overline{v_{m+2}}, \dots, \overline{v_n}$ are linearly independent

Hence they are basis of V/W

<u>Proposition</u>:- Let $T: V \to V'$ be a L.T then $\ker T$ is a subspace of V

<u>proof</u>: $W = \ker T = \{v / v \in V, T(v) = 0\}$

Let $v_1, v_2 \in W = \ker T$

then $T(v_1) = 0$ and $T(v_2) = 0$

Consider, $T(v_1 - v_2) = T(v_1) - T(v_2)$ (since T is LT)

$$T(v_1 - v_2) = 0$$

$$\Rightarrow v_1 - v_2 \in W$$

$\therefore$ $W = \ker T$ is an abelian subgroup of V

$$T(av) = a\,T(v) \quad (L.T)$$

$$= a\,0$$

$$= 0$$

$\Rightarrow av \in W$

$\therefore$ $T = \ker T$ is a subspace of V

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Subject : Mathematics

Assignment on : Polar Co-ordinates and Number theory

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Theoram: The linear diaphentine equation $ax+by=c$ has a solution iff $d|c$, where $d=gcd(a,b)$. If $x_0\ y_0$ is any perticular soln of the equation then all other equation are given by $x=x_0+\frac{b}{d}t$, $y=y_0-\frac{a}{d}t$, where t is an arbitrary integer.

Proof :- Suppose $ax+by=c$ has a solution then $ax_0+by_0=c$ for suitable integer x_0 & y_0

Since $d=gcd(a,b)$

$\therefore d|a$, $d|b$

$\Rightarrow a=dr$ —①, $b=ds$ —② where r & s are Integers.

Now, $c=ax_0+by_0$

$=drx_0+dsy_0$

$c=d(\underbrace{rx_0+sy_0}_{\text{integers}})$

$\Rightarrow d|c$

Conversly, suppose $d|c$ then $c=dt$, wher t is an integer.

Since $d=gcd(a,b)$

$\therefore$ there exist integers x_0 & y_0 such that $d=ax_0+by_0$

$\therefore c=dt=(ax_0+by_0)t=ax_0t+by_0t$

then $x=x_0t$, $y=y_0t$ is a posible solution of $ax+by=c$

$\therefore ax+by=c$ has a solution iff $d|c$.

To establish second part of theorum,

Let (x_0,y_0) be a perticular solution of $ax+by=c$ and (x',y') be any other solution

then $ax_0+by_0=c$

$ax'+by'=c$

$\therefore ax_0 + by_0 = ax' + by'$

$b(y_0 - y') = a(x' - x_0)$

$a(x' - x_0) = b(y_0 - y') \quad -③$

we know that, if $\gcd(a,b) = d$ then $\gcd\left(\frac{a}{b}, \frac{b}{d}\right) = 1$

$\therefore$ there exist integers $r = \frac{a}{d}$, $s = \frac{b}{d}$ such that $\gcd(r,s) = 1$

i.e. $a = rd$, $b = sd$

Substitute these value of a & b in ③

$rd(x' - x_0) = sd(y_0 - y')$

Since $d > 0$, canceling d

$r(x' - x_0) = s(y_0 - y') \quad —④$

Since $r \mid r(x' - x_0)$ $\quad \therefore r \mid s(y_0 - y')$

$r \mid s(y_0 - y')$ and $\gcd(r,s) = 1 \Rightarrow r \mid y_0 - y'$

[By Euclides lemma: $a \mid bc$, $\gcd(a,b) = 1 \Rightarrow a \mid c$]

$\Rightarrow y_0 - y' = rt \quad -⑤$, t is an integer

$y' = y_0 - rt$

$\therefore y' = y_0 - \frac{a}{d} t \qquad (\because r = \frac{a}{d})$

from ⑤, $y_0 - y' = rt$

Substitute ⑤ in ④

$r(x' - x_0) = s(rt)$

$x' - x_0 = st$

$\therefore x_0 + st = x'$

$x' = x_0 + \frac{b}{d} t \qquad (\because s = \frac{b}{d})$

x' and y' satisfies the equation $ax + by = c$

For LHS $ax' + by' = a\left(x_0 + \frac{b}{d} t\right) + b\left(y_0 - \frac{a}{d} t\right)$

$ax' + by' = ax_0 + by_0 = c = R.H.S$

Hence the given diaphantine equation has an infinite number of solution, one for each value of t.

Find all the solution of linear diaphentine equation $172x+20y=1000$ also obtain +ve integer solution, If it exists.

$172x+20y=1000$ —①

First we find gcd (172, 20) using Euclidean algorithm,

$172 = (8)20 + 12$ —②

$20 = (1)12 + 8$ —③

$12 = (1)8 + 4$ —④

$8 = (2)4 + 0$

$\therefore$ gcd (172, 20) = 4, last non zero remainder.

gcd (172, 20) = $4 | 1000$

$\therefore$ given L.D.E has a solution.

$4 = 12 - (1)8$ (from ④)

$4 = 12 - (1)[20 - (1)12]$ (from ③)

$4 = (2)12 - (1)20$

$= (2)[172 - (8)20] - (1)20$

$4 = (2)172 + (-17)20$ —⑤

Compare ⑤ with ① multiply 250 throught the equation ⑤, we get

$(500)172 + (-4250)20 = 1000$

$\therefore x_0 = 500$, $y_0 = -4250$ gives one solution for equation ①

$\therefore$ All solutions are given by,

$x = x_0 + \frac{b}{d}t$, $y = y_0 - \frac{a}{d}t$

$x = 500 + 5t$, $y = -4250 - 43t$ —⑥ where t is an integer.

To find +ve integer solution:

$x = 500 + 5t > 0$ & $y = -4250 - 43t > 0$

$\Rightarrow 5t > -500$ & $43t < -4250$

$t > -100$ & $t < -98.837$

$\Rightarrow -100 < t < -98.837$

$\therefore t = -99$ (integer)

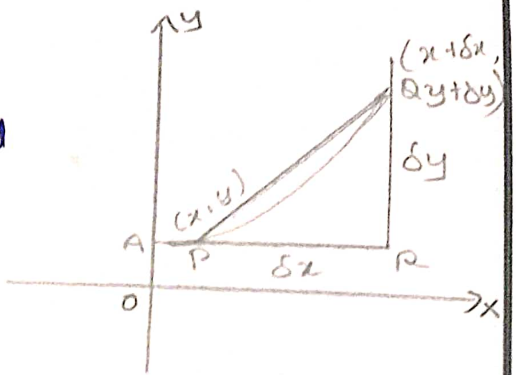
$\therefore x = 5$ & $y = 7$ (from ⑥)

$\therefore$ equation ① has a unique position solution

$x = 5$ & $y = 7$

Derive differential coefficient of length of an arc.

Let us consider the curve to be $y = f(x)$.



A is some fixed point upon the curve and P any other point on the curve whose co-ordinates are (x, y).

Let 's' be the length of an arc AP that is the distance along the curve from A to P and δs be the increasing the length of an arc AP that is δy when x & y are increased by δx & δy.

Let Q be the point is coordinates are $(x + \delta x), (y + \delta y)$ and an arc PQ be δs.

Now,
$$\frac{\delta s}{\delta x} = \frac{\delta s}{\delta x} \cdot \frac{\text{chord } PQ}{\text{arc } PQ}$$
$$= \frac{\delta s}{\text{arc} PQ} \cdot \frac{\text{chord } PQ}{\delta x}$$

Now chord $PQ = \sqrt{(PR)^2 + (RQ)^2} = \sqrt{(\delta x)^2 + (\delta y)^2}$

Then,
$$\frac{\delta s}{\delta x} = \frac{\delta s}{\text{arc} PQ} \cdot \frac{\sqrt{(\delta x)^2 + (\delta y)^2}}{\delta x}$$
$$= \frac{\delta s}{\text{arc } PQ} \cdot \sqrt{\frac{(\delta x)^2 + (\delta y)^2}{(\delta x)^2}}$$
$$= \frac{\delta s}{\text{arc} PQ} \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2}$$

$$\therefore \lim_{\delta x \to 0} \frac{\delta s}{\delta x} = \lim_{\delta x \to 0} \frac{\delta s}{\text{arc} PQ} \cdot \lim_{\delta x \to 0} \sqrt{1 + \left(\frac{\delta y}{\delta x}\right)^2}$$

But $\frac{\delta s}{\text{arc} PQ} = 1$ where $\delta x \to 0$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \frac{dy}{dx} \qquad \lim_{\delta x \to 0} \frac{\delta s}{\delta x} = \frac{ds}{dx}$$

Then ① becomes
$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

$$\frac{ds}{dy} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$$

Find the pedal equation of the curve $r^m = a^m \cos m\theta$.

Given, $r^m = a^m \cos m\theta$

taking log on both side

$$\log r^m = \log (a^m \cos m\theta)$$

$$m \log r = \log a^m + \log \cos m\theta$$

$$m \log r = m \log a + \log \cos m\theta$$

$$m \cdot \frac{1}{r} \cdot \frac{dr}{d\theta} = 0 + \frac{1}{\cos m\theta} (-\sin m\theta) \cdot m$$

$$\frac{m}{r} \frac{dr}{d\theta} = \frac{m \sin m\theta}{\cos m\theta} = -\tan m\theta$$

$$\frac{1}{r} \frac{dr}{d\theta} = -\tan m\theta$$

$$\cot \phi = \cot (\pi/2 + m\theta)$$

$$\phi = \cot^{-1} \cot (\pi/2 + m\theta)$$

$$\phi = \pi/2 + m\theta$$

$$p = r \sin \phi$$

$$p = r \sin (\pi/2 + m\theta)$$

$$p = r \cos m\theta$$

$$= r \cdot \frac{r^m}{a^m} = \frac{r^{m+1}}{a^m}$$

Find the perpendicular length from the pole on the tangent to the curve $r(\theta - 1) = a\theta^2$

Solution: we have $r(\theta - 1) = a\theta^2$

$$\Rightarrow r = \frac{a\theta^2}{(\theta - 1)}$$

length of the perpendicular from the to the tangent is given by $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

then $$\frac{dr}{d\theta} = \frac{(\theta - 1) 2a\theta - a\theta^2 (1)}{(\theta - 1)^2}$$

$$= \frac{2a\theta (\theta - 1) - a\theta^2}{(\theta - 1)^2}$$

$$\frac{dr}{d\theta} = \frac{2a\theta^2 - 2a\theta - a\theta^2}{(\theta-1)^2}$$

$$= \frac{a\theta^2 - 2a\theta}{(\theta-1)^2}$$

$$= \frac{a\theta(\theta-2)}{(\theta-1)^2}$$

$$\frac{1}{p^2} = \frac{1}{\left(\frac{a\theta}{\theta-1}\right)^2} + \frac{1}{\left(\frac{a\theta}{\theta-1}\right)^4}\left(\frac{a\theta(\theta-2)}{(\theta-1)^2}\right)^2$$

$$= \frac{1}{\frac{a^2\theta^4}{(\theta-1)^2}} + \frac{1}{\frac{a^4\theta^8}{(\theta-1)^4}}\left(\frac{a^2\theta^2(\theta-2)^2}{(\theta-1)^4}\right)$$

$$= \frac{(\theta-1)^2}{a^2\theta^4} + \frac{\cancel{(\theta-1)^4}}{a^4\theta^8}\left(\frac{a^2\theta^2(\theta-2)^2}{\cancel{(\theta-1)^4}}\right)$$

$$= \frac{(\theta-1)^2}{a^2\theta^4} + \frac{(\theta-2)^2}{a^2\theta^6}$$

$$= \frac{1}{a^2\theta^4}\left((\theta-1)^2 + \frac{(\theta-2)^2}{\theta^2}\right)$$

$$= \frac{1}{a^2\theta^4}\left(\frac{\theta^2(\theta-1)^2 + (\theta-2)^2}{\theta^2}\right)$$

$$= \frac{1}{a^2\theta^4}\left(\frac{\theta^2(\theta^2+1-2\theta) + (\theta^2+4-4\theta)}{\theta^2}\right)$$

$$= \frac{1}{a^2\theta^4}\left(\frac{\theta^4+\theta^2-2\theta^3+\theta^2+4-4\theta}{\theta^2}\right)$$

$$= \frac{1}{a^2\theta^4}\left(\frac{\theta^4+2\theta^2-2\theta^3-4\theta+4}{\theta^2}\right)$$

$$= \frac{1}{a^2}\left(\frac{\theta^4}{\theta^6} + \frac{2\theta^2}{\theta^6} - \frac{2\theta^3}{\theta^6} - \frac{4\theta}{\theta^6} + \frac{4}{\theta^6}\right)$$

$$\frac{1}{p^2} = \frac{1}{a^2}\left(\frac{1}{\theta^2} + \frac{2}{\theta^4} - \frac{2}{\theta^3} - \frac{4}{\theta^5} + \frac{4}{\theta^6}\right)$$

1. Write the formula ~~o~~for polar sub tangent

$$r^2 \frac{d\theta}{dr} = -\frac{d\theta}{du} \quad \text{where } u = \frac{1}{r}$$

2. Mention the pol formula of Derivative

$$\frac{ds}{d\theta} = \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$$

$$\frac{ds}{dr} = \sqrt{1 + r^2\left(\frac{d\theta}{dr}\right)^2}$$

3. Calculate $\frac{ds}{dt}$ for the curves $x = t^2$, $y = t-1$

We have $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$$\frac{dx}{dt} = 2t, \quad \frac{dy}{dt} = 1$$

$$\frac{ds}{dt} = \sqrt{(2t)^2 + 1}$$

$$= \sqrt{4t^2+1}$$